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# Algorithm for Computing the Propagator for Higher Derivative Gravity Theories

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A simple algorithm for computing the propagator for higher derivative gravity theories based on the Barnes–Rivers operators is presented. The prescription is used, among other things, to obtain the propagator for quadratic gravity in an unconventional gauge. We also find the propagator for both gravity and quadratic gravity in an interesting gauge recently baptized the "Einstein" gauge [Hitzer and Dehnen, *Int. J. Theor. Phys.* **36** (1997), 559].

## 1. INTRODUCTION

The usual way of computing the propagator for gravity theories is based on the following standard procedures.

1. Linearize the Lagrangian corresponding to the original theory. This is done in two steps. First we decompose the metric $g_{\mu\nu}$ as

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} \tag{1}$$

where $\eta_{\mu\nu}$ is the usual Minkowski metric and $\kappa$ is the square root of Einstein's constant, and afterward we insert (1) into the gravitational Lagrangian. Let us designate the resulting Lagrangian as $\mathcal{L}_g$.

2. Add to $\mathcal{L}_g$ a suitable gauge-fixing Lagrangian $\mathcal{L}_{gf}$. Of course, we are assuming that the theory in hand is gauge-invariant.

3. Cast the resulting Lagrangian,

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$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_{gf}$$

into the bilinear form

$$\mathcal{L} = \tfrac{1}{2} h^{\mu\nu}\, \mathbb{O}_{\mu\nu,\rho\sigma}\, h^{\rho\sigma}$$

4. Invert the operator $\mathbb{O}$.

The last item in the list above, i.e., the inversion of the gravitational kinetic matrix which is necessary to calculate the graviton propagator, involves in most cases a substantial amount of Lorentz algebra on symmetric rank-two tensors. To avoid this tedious calculation and to save time, we propose an algorithm for inverting the operator $\mathbb{O}$ based on the Barnes–Rivers operators [1–5]. To prove the efficacy of the prescription, we compute the propagator concerning quadratic gravity using an unconventional gauge-fixing Lagrangian. From this result we obtain in a straightforward way the propagator for both general relativity and higher derivative gravity in a series of interesting gauges. The "Einstein" gauge [6] is considered as well. We also calculate the propagator for a higher derivative gravity theory which is not gauge-invariant, namely, Fierz–Pauli higher derivative gravity [7].

We use natural units throughout. In our convention the signature is $(+- --)$. The curvature tensor is defined by $R^{\alpha}{}_{\beta\gamma\delta} = -\partial_\delta \Gamma^{\alpha}{}_{\beta\gamma} + \ldots$, the Ricci tensor by $R_{\mu\nu} = R^{\alpha}{}_{\mu\nu\alpha}$, and the curvature scalar by $R = g^{\mu\nu} R_{\mu\nu}$, where $g_{\mu\nu}$ is the metric tensor.

## 2. THE ALGORITHM

As we have already mentioned, to find the graviton propagator we have to invert the operator $\mathbb{O}_{\mu\nu,\rho\sigma}$. This operator is symmetric in the indices $(\mu\nu)$, $(\rho\sigma)$ and under the interchange of $\mu\nu$ with $\rho\sigma$. Barnes and Rivers showed that a complete set of operators that span the space of the operators with the symmetries above is given in momentum space by [1–5]

$$\begin{aligned}
P^1_{\mu\nu,\rho\sigma} &= \tfrac{1}{2}(\Theta_{\mu\rho}\omega_{\nu\sigma} + \Theta_{\mu\sigma}\omega_{\nu\rho} + \Theta_{\nu\rho}\omega_{\mu\sigma} + \Theta_{\nu\sigma}\omega_{\mu\rho})\\
P^2_{\mu\nu,\rho\sigma} &= \tfrac{1}{2}(\Theta_{\mu\rho}\Theta_{\nu\sigma} + \Theta_{\mu\sigma}\Theta_{\nu\rho}) - \tfrac{1}{3}\Theta_{\mu\nu}\Theta_{\rho\sigma}\\
P^0_{\mu\nu,\rho\sigma} &= \tfrac{1}{3}\Theta_{\mu\nu}\Theta_{\rho\sigma}\\
\overline{P}^0_{\mu\nu,\rho\sigma} &= \omega_{\mu\nu}\omega_{\rho\sigma}\\
\overline{\overline{P}}^0_{\mu\nu,\rho\sigma} &= \Theta_{\mu\nu}\omega_{\rho\sigma} + \omega_{\mu\nu}\Theta_{\rho\sigma}
\end{aligned}$$

where $\Theta_{\mu\nu}$ and $\omega_{\mu\nu}$ are the usual longitudinal and transverse vector projection operators

$$\Theta_{\mu\nu} = \eta_{\mu\nu} - k_\mu\, k_\nu/k^2, \qquad \omega_{\mu\nu} = k_\mu\, k_\nu/k^2$$

which satisfy the relations

$$\Theta_{\mu\rho}\Theta^\rho{}_\nu = \Theta_{\mu\nu}, \qquad \omega_{\mu\rho}\omega^\rho{}_\nu = \omega_{\mu\nu}, \qquad \Theta_{\mu\rho}\omega^\rho{}_\nu = 0$$

The Minkowski metric in our convention is $\eta_{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)$. Here $k_\mu$ is the momentum of the graviton exchanged.

The set of the four operators $\{P^1, P^2, P^0, \overline{P}^0\}$ is a complete set of projection operators for symmetric rank-two tensors. They are idempotent, mutually orthogonal, and satisfy the completeness relation

$$[P^1 + P^2 + P^0 + \overline{P}^0]_{\mu\nu,\rho\sigma} = \tfrac{1}{2}(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho}) \equiv I_{\mu\nu,\rho\sigma}$$

In the rest frame of a massive tensor field, the family of operators $\{P^1, P^2, P^0, \overline{P}^0\}$ select out the spin-one, spin-two, and two spin-zero parts of the field. However, in order to have a complete basis for the operator space of the gravitational field equations, we must include in that collection the transfer operator $\overline{\overline{P}}{}^0$. Its multiplicative table is

$$\overline{\overline{P}}{}^0 P^1 = P^1\overline{\overline{P}}{}^0 = \overline{\overline{P}}{}^0 P^2 = P^2\overline{\overline{P}}{}^0 = O$$
$$\overline{\overline{P}}{}^0\overline{\overline{P}}{}^0 = 3(P^0 + \overline{P}^0)$$
$$P^0\overline{\overline{P}}{}^0 = \overline{\overline{P}}{}^0\overline{P}^0 = P^{\theta\omega}$$
$$\overline{P}^0\overline{\overline{P}}{}^0 = \overline{\overline{P}}{}^0 P^0 = P^{\omega\theta}$$

where $P^{\theta\omega}{}_{\mu\nu,\rho\sigma} \equiv \theta_{\mu\nu}\omega_{\rho\sigma}$, $P^{\omega\theta}{}_{\mu\nu,\rho\sigma} \equiv \omega_{\mu\nu}\theta_{\rho\sigma}$, and $O$ is the null operator. Note that $\overline{\overline{P}}{}^0 = P^{\theta\omega} + P^{\omega\theta}$.

We are now ready to find the propagator. Expanding both operators $\mathbb{O}$ and $\mathbb{O}^{-1}$ in the basis $\{P^1, P^2, P^0, \overline{P}^0, \overline{\overline{P}}{}^0\}$, we get

$$\mathbb{O} = x_1 P^1 + x_2 P^2 + x_0 P^0 + \bar{x}_0 \overline{P}^0 + \bar{\bar{x}}_0 \overline{\overline{P}}{}^0$$
$$\mathbb{O}^{-1} = y_1 P^1 + y_2 P^2 + y_0 P^0 + \bar{y}_0 \overline{P}^0 + \bar{\bar{y}}_0 \overline{\overline{P}}{}^0$$

Taking into account that $\mathbb{O}\mathbb{O}^{-1} = I = P^1 + P^2 + P^0 + \overline{P}^0$, we promptly obtain the following set of simultaneous equations:

$$x_1 y_1 = 1, \qquad \bar{x}_0\bar{y}_0 + 3\bar{\bar{x}}_0\bar{\bar{y}}_0 = 1$$
$$x_2 y_2 = 1, \qquad \bar{\bar{x}}_0 y_0 + \bar{x}_0\bar{\bar{y}}_0 = 0$$
$$x_0 y_0 + 3\bar{\bar{x}}_0\bar{\bar{y}}_0 = 1, \qquad \bar{\bar{x}}_0\bar{y}_0 + x_0\bar{\bar{y}}_0 = 0$$

Row reducing the augmented matrix of the system to echelon form yields

$$
\begin{bmatrix}
x_1 & 0 & 0 & 0 & 0 & 1 \\
0 & x_2 & 0 & 0 & 0 & 1 \\
0 & 0 & x_0 & 0 & 3\bar{\bar{x}}_0 & 1 \\
0 & 0 & \bar{\bar{x}}_0 & 0 & \bar{x}_0 & 0 \\
0 & 0 & 0 & \bar{x}_0 & 3\bar{\bar{x}}_0 & 1 \\
0 & 0 & 0 & \bar{\bar{x}}_0 & x_0 & 0
\end{bmatrix}
$$

$$
\sim \begin{bmatrix}
x_1 & 0 & 0 & 0 & 0 & 1 \\
0 & x_2 & 0 & 0 & 0 & 1 \\
0 & 0 & x_0 & 0 & 3\bar{\bar{x}}_0 & 1 \\
0 & 0 & 0 & \bar{x}_0 & 3\bar{\bar{x}}_0 & 1 \\
0 & 0 & 0 & 0 & (x_0\bar{x}_0 - 3\bar{\bar{x}}_0^2) & -\bar{\bar{x}}_0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

Therefore, the propagator is given by

$$
\begin{aligned}
\mathbb{O}^{-1} = \frac{1}{x_1} P^1 + \frac{1}{x_2} P^2 + \frac{\bar{x}_0}{x_0\bar{x}_0 - 3\bar{\bar{x}}_0^2} P^0 + \frac{x_0}{x_0\bar{x}_0 - 3\bar{\bar{x}}_0^2} \bar{P}^0 \\
- \frac{\bar{\bar{x}}_0}{x_0\bar{x}_0 - 3\bar{\bar{x}}_0^2} \bar{\bar{P}}^0
\end{aligned} \tag{2}
$$

The expansion of the operator $\mathbb{O}$ in the basis $\{P^1, P^2, P^0, \bar{P}^0, \bar{\bar{P}}^0\}$ is trivially obtained by the use of the following tensorial identities, which follow easily from the very definition of the operators $P^1$, $P^2$, $P^0$, $\bar{P}^0$, $\bar{\bar{P}}^0$:

$$
\begin{aligned}
\tfrac{1}{2}(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho}) &= [P^1 + P^2 + P^0 + \bar{P}^0]_{\mu\nu,\rho\sigma} \\
\eta_{\mu\nu}\eta_{\rho\sigma} &= [3P^0 + \bar{P}^0 + \bar{\bar{P}}^0]_{\mu\nu,\rho\sigma} \\
\eta_{\mu\rho}k_\nu k_\sigma + \eta_{\mu\sigma}k_\nu k_\rho + \eta_{\nu\rho}k_\mu k_\sigma + \eta_{\nu\sigma}k_\mu k_\rho &= k^2[2P^1 + 4\bar{P}^0]_{\mu\nu,\rho\sigma} \\
\eta_{\mu\nu}k_\rho k_\sigma + \eta_{\rho\sigma}k_\mu k_\nu &= k^2[\bar{\bar{P}}^0 + 2\bar{P}^0]_{\mu\nu,\rho\sigma} \\
k_\mu k_\nu k_\rho k_\sigma &= k^4\bar{P}^0_{\mu\nu,\rho\sigma}
\end{aligned} \tag{3}
$$

The identities

$$
\begin{aligned}
P^2_{\mu\nu,\rho\sigma} &= \tfrac{1}{2}[\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho}] - \tfrac{1}{3}\eta_{\mu\nu}\eta_{\rho\sigma} - [P^1 + \tfrac{2}{3}\bar{P}^0 - \tfrac{1}{3}\bar{\bar{P}}^0]_{\mu\nu,\rho\sigma} \\
P^0_{\mu\nu,\rho\sigma} &= \tfrac{1}{3}\eta_{\mu\nu}\eta_{\rho\sigma} - \tfrac{1}{3}[\bar{P}^0 + \bar{\bar{P}}^0]_{\mu\nu,\rho\sigma}
\end{aligned}
$$

greatly facilitate the task of costing the propagator in a form where the terms proportional to the graviton momentum are omitted, which in practice widely simplifies computations involving conserved currents.

## 3. THE PROPAGATOR FOR QUADRATIC GRAVITY IN AN UNCONVENTIONAL GAUGE

The Lagrangian for quadratic gravity is given by

$$\overline{\mathcal{L}}_g = \left[\frac{2R}{\kappa^2} + \frac{\alpha}{2} R^2 + \frac{\beta}{2} R^2_{\mu\nu}\right]\sqrt{-g} \tag{4}$$

where $\kappa^2 = 32\pi G$, $G$ being Newton's constant, and $\alpha$ and $\beta$ are dimensionless parameters. Linearizing (4), we obtain

$$\begin{aligned}\mathcal{L}_g = \frac{b}{4}\,&[\Box h_{\mu\nu}\,\Box h^{\mu\nu} - (A^{\mu}{}_{,\mu})^2 - F^2_{\mu\nu} + (1 + 4c)(A^{\mu}{}_{,\mu} - \Box\phi)^2] \\ &- \frac{1}{2}\,[h_{\mu\nu}\,\Box h^{\mu\nu} + A^2_{\nu} + (A_{\nu} - \partial_{\nu}\phi)^2]\end{aligned} \tag{5}$$

where $A^{\mu} \equiv h^{\mu\nu}{}_{,\nu}$, $\phi \equiv h$, $F_{\mu\nu} \equiv A_{\mu,\nu} - A_{\nu,\mu}$, $b \equiv \beta\kappa^2/2$, $c \equiv \alpha/\beta$. Indices are lowered (raised) using $\eta_{\mu\nu}$ ($\eta^{\mu\nu}$).

Lagrangian (5) is invariant under the infinitesimal coordinate transformation

$$x^{\mu} \to x^{\mu} + \kappa\xi^{\mu}(x)$$

where $\xi^{\mu}(x)$ is an infinitesimal vector field. It must be infinitesimal to avoid inconsistency with (1). Under this transformation we have, from (1),

$$h_{\mu\nu}(x) \to h_{\mu\nu}(x) - \xi_{\mu,\nu} - \xi_{\nu,\mu} \tag{6}$$

The presence of the local gauge symmetry (6) requires the addition of a gauge-fixing term $\mathcal{L}_{gf}$ to the Lagrangian (5). It is common practice to choose a linear combination of $A_{\mu}$ and $\partial_{\mu}\phi$ as gauge functions. However, looking at (5), we clearly see the presence not only of this linear combination, but also of its curl ($F_{\mu\nu}$) and its divergence ($A^{\mu}{}_{,\mu} - \Box\phi$). Therefore, we choose the gauge-fixing Lagrangian

$$\mathcal{L}_{gf} = \lambda_1(A_{\nu} - \lambda\partial_{\nu}\phi)^2 + \frac{b}{4}\,[\lambda_2(A^{\mu}{}_{,\mu} - \lambda\Box\phi)^2 + \lambda_3 F^2_{\mu\nu}]$$

which, despite being rather unconventional, it is very convenient for our purposes.

Casting the Lagrangian

$$\mathscr{L} = \mathscr{L}_g + \mathscr{L}_{gf}$$

into the bilinear form

$$\mathscr{L} = \frac{1}{2} h^{\mu\nu}\mathbb{O}_{\mu\nu,\rho\sigma}h^{\rho\sigma}$$

and expanding the operator $\mathbb{O}$ in the basis $\{P^1, P^2, P^0, \overline{P}^0, \overline{\overline{P}}^0\}$ with the help of the tensorial identities (3), we obtain

$$\mathbb{O} = x_1 P^1 + x_2 P^2 + x_0 P^0 + \overline{x}_0 \overline{P}^0 + \overline{\overline{x}}_0 \overline{\overline{P}}^0$$

where

$$x_1 \equiv b/2(\lambda_3 k^4 + 2\lambda_1 k^2/b)$$
$$x_2 \equiv b/2(k^4 + 2k^2/b)$$
$$x_0 \equiv b/2(4k^4 - 4k^2/b + 12k^4 c + 3\lambda_2\lambda^2 k^4 + 12\lambda_1\lambda^2 k^2/b)$$
$$\overline{x}_0 \equiv b/2(\lambda_2 k^4 - 2\lambda\lambda_2 k^4 - 8\lambda\lambda_1 k^2/b + \lambda_2\lambda^2 k^4 + 4\lambda_1\lambda^2 k^2/b + 4\lambda_1 k^2/b)$$
$$\overline{\overline{x}}_0 \equiv b/2(-\lambda\lambda_2 k^4 - 4\lambda\lambda_1 k^2/b + \lambda_2\lambda^2 k^4 + 4\lambda_1\lambda^2 k^2/b)$$

The propagator in momentum space is given by (2). From this result we can obtain the propagator in a series of interesting gauges not only for higher derivative gravity, but also for Einstein's gravity, by judiciously choosing the parameters $\lambda$, $\lambda_1$, $\lambda_2$, and $\lambda_3$. We list below the most important covariant gauges that result from such choices.

1. *Julve-Tonin gauge* ($\lambda = 0$) [8]:

$$\mathscr{L}_{gf} = \lambda_1 A_\nu^2 + \frac{b}{4}[\lambda_2(A^\mu{}_{,\mu})^2 + \lambda_3 F_{\mu\nu}^2]$$

Propagator:

$$\mathbb{O}^{-1} = \frac{m_1^2}{k^2(m_1^2\lambda_1 - k^2\lambda_3)} P^1 + \frac{m_1^2}{k^2(m_1^2 - k^2)} P^2 + \frac{m_0^2}{2k^2(k^2 - m_0^2)} P^0 + \frac{m_1^2}{k^2(2\lambda_1 m_1^2 - \lambda_2 k^2)} \overline{P}^0$$

where

$$m_0^2 \equiv \frac{2}{\kappa^2(3\alpha + \beta)}, \qquad m_1^2 \equiv -\frac{4}{\kappa^2\beta}$$

Absence of tachyons requires $\beta < 0$ and $3\alpha + \beta > 0$. Note that the choice

$\lambda = 0$ gives a propagator that only contains the spin-projection operators, i.e., $P^1$, $P^2$, $P^0$, $\overline{P}^0$, and it gives a propagator all parts of which behave like $k^{-4}$.

For Einstein's gravity, we have

$$\mathcal{L}_{gf} = \lambda_1 A_\nu^2$$

$$\mathbb{O}^{-1} = \frac{1}{\lambda_1 k^2} P^1 + \frac{1}{k^2} P^2 - \frac{1}{2k^2} P^0 + \frac{1}{2\lambda_1 k^2} \overline{P}^0$$

2. *de Donder gauge* ($\lambda_2 = \lambda_3 = 0$, $\lambda = 1/2$):

$$\mathcal{L}_{gf} = \lambda_1 \left( A_\nu - \frac{1}{2} \partial_\nu \phi \right)^2$$

Propagator:

$$\mathbb{O}^{-1} = \frac{1}{\lambda_1 k^2} P^1 + \frac{m_1^2}{k^2(m_1^2 - k^2)} P^2 + \frac{m_0^2}{2k^2(k^2 - m_0^2)} P^0 + \left[ \frac{2}{\lambda_1 k^2} + \frac{3m_0^2}{2k^2(k^2 - m_0^2)} \right] \overline{P}^0 + \frac{m_0^2}{2k^2(k^2 - m_0^2)} \overline{\overline{P}}^0$$

For gravitation, the propagator in the de Donder gauge can be expressed as

$$\mathbb{O}^{-1} = \frac{1}{\lambda_1 k^2} P^1 + \frac{1}{k^2} P^2 - \frac{1}{2k^2} P^0 + \left( \frac{2}{\lambda_1 k^2} - \frac{3}{2k^2} \right) \overline{P}^0 - \frac{1}{2k^2} \overline{\overline{P}}^0$$

3. *Feynman gauge* ($\lambda_2 = \lambda_3 = 0$, $\lambda_1 = 1$, $\lambda = 1/2$):

$$\mathcal{L}_{gf} = \left( A_\nu - \frac{1}{2} \partial_\nu \phi \right)^2$$

Propagator:

$$\mathbb{O}^{-1} = \frac{1}{k^2} P^1 + \frac{m_1^2}{k^2(m_1^2 - k^2)} P^2 + \frac{m_0^2}{2k^2(k^2 - m_0^2)} P^0 + \left[ \frac{2}{k^2} + \frac{3m_0^2}{2k^2(k^2 - m_0^2)} \right] \overline{P}^0 + \frac{m_0^2}{2k^2(k^2 - m_0^2)} \overline{\overline{P}}^0$$

For Einstein's gravity, the propagator is given in the Feynman gauge by

$$\mathcal{O}^{-1}_{\mu\nu,\rho\sigma} = \frac{\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}}{2k^2}$$

## 4. THE PROPAGATOR IN THE "EINSTEIN" GAUGE

To find the propagator for quadratic gravity in the "Einstein" gauge [6], we add to (5) the gauge-fixing Lagrangian,

$$\mathcal{L}_{gf} = \frac{\lambda\phi^2}{\kappa^2}$$

where $\lambda$ is the gauge parameter, and afterward we cast the resulting Lagrangian $\mathcal{L}$ into the bilinear form $\mathcal{L} = 1/2h^{\mu\nu}\mathcal{O}_{\mu\nu,\rho\sigma}h^{\rho\sigma}$. Expanding the operator $\mathcal{O}$ in the basis $\{P^1, P^2, P^0, \overline{P}^0, \overline{\overline{P}}^0\}$, we get

$$\mathcal{O} = \frac{b}{2}\left(k^4 + \frac{2k^2}{b}\right)P^2 + \frac{b}{2}\left(4k^4 - \frac{4k^2}{b} + 12k^4c + \frac{12\lambda}{bk^2}\right)P^0$$
$$+ \frac{2\lambda}{\kappa^2}\,\overline{P}^0 + \frac{2\lambda}{\kappa^2}\,\overline{\overline{P}}^0$$

As a consequence, the propagator is given in momentum space by

$$\mathcal{O}^{-1} = \frac{m_1^2}{k^2(m_1^2 - k^2)}\,P^2 + \frac{m_0^2}{2k^2(k^2 - m_0^2)}\,P^0$$
$$+ \left[\frac{\kappa^2}{2\lambda} + \frac{3m_0^2}{2k^2(k^2 - m_0^2)}\right]\overline{P}^0 - \frac{m_0^2}{2k^2(k^2 - m_0^2)}\,\overline{\overline{P}}^0$$

If we assume that $m_0$ and $m_1$ are real, which corresponds to the absence of tachyons (both positive and negative energy) in the dynamical field, it is easy to see that the previous expression tends to

$$\mathcal{O}^{-1} = \frac{1}{k^2}\,P^2 - \frac{1}{2k^2}\,P^0 + \left[\frac{\kappa^2}{2\lambda} - \frac{3}{2k^2}\right]\overline{P}^0 + \frac{1}{2k^2}\,\overline{\overline{P}}^0$$

as both $m_0^2$ and $m_1^2 \rightarrow \infty$, which is the propagator for Einstein's gravity in the "Einstein" gauge.

## 5. THE PROPAGATOR FOR A HIGHER DERIVATIVE GRAVITY THEORY WHICH IS NOT GAUGE-INVARIANT

We consider now Fierz–Pauli higher derivative gravity [7]. To arrive at the Lagrangian for this theory, we add the linear part of the Lagrangian containing the four-derivative terms $(\alpha/2)R^2\sqrt{-g}$ and $(\beta/2)R^2_{\mu\nu}\sqrt{-g}$, namely

$$\mathcal{L}_{hd} = \frac{b}{4}\left[\Box h_{\mu\nu}\,\Box h^{\mu\nu} - (A^{\mu}{}_{,\mu})^2 - F^2_{\mu\nu} + (1 + 4c)(A^{\mu}{}_{,\mu} - \Box\phi)^2\right]$$

where $b \equiv \beta\tilde{\kappa}^2/2$, $c \equiv \alpha/\beta$, $\tilde{\kappa}^2$ being the "Einstein's constant" for Fierz–Pauli gravity [9], to the Fierz–Pauli Lagrangian, i.e.,

$$\mathcal{L}_{\mathrm{FP}} = -\frac{1}{2}\left[h^{\mu\nu}\Box h_{\mu\nu} + (A_{\nu})^2 + (A_{\nu} - \partial_{\nu}\phi)^2\right] - \frac{1}{2}m^2(h^2_{\mu\nu} - \phi^2) \qquad (7)$$

Since this theory is not gauge-invariant owing to the Proca-like mass term, we do not need to introduce any gauge fixing-term into (7) in order to find the propagator. As a result, all we have to do in this case is to invert the operator

$$\mathbb{O} = \left(\frac{b}{2}k^4 + k^2 - m^2\right)P^2 - m^2P^1 + (2bk^4 - 2k^2 + 6bck^4 + 2m^2)P^0 + m^2\overline{\overline{P}}{}^0$$

From (2) we promptly obtain

$$\mathbb{O}^{-1} = \frac{1}{(b/2)k^4 + k^2 - m^2}P^2 - \frac{1}{m^2}P^1 - \frac{2bk^4 + 6bck^4 - 2k^2 + 2m^2}{3m^4}\overline{P}{}^0 + \frac{1}{3m^2}\overline{\overline{P}}{}^0 \qquad (8)$$

If we take $b = c = 0$ in (8), we recover the propagator concerning Fierz–Pauli gravity [10], namely

$$\mathbb{O}^{-1}_{\mu\nu,\rho\sigma} = \frac{\frac{1}{2}(\eta_{\mu\rho}\,\eta_{\nu\sigma} + \eta_{\mu\sigma}\,\eta_{\nu\rho}) - \frac{1}{3}\eta_{\mu\nu}\,\eta_{\rho\sigma}}{k^2 - m^2}$$

where we have omitted the terms proportional to the graviton momentum.

## 6. CONCLUSIONS

We proposed a prescription for finding the propagator concerning higher derivative gravity theories based on the Barnes–Rivers operators. Using this algorithm, we computed the propagator for quadratic gravity in an unconventional gauge and, by a suitable choice of the gauge parameters, we reobtained the propagator in a series of interesting gauges which are broadly used in the literature. We also calculated the propagator for both quadratic gravity and Einstein's gravity in a very curious gauge baptized the "Einstein" gauge by Hitzer and Dehmen [6]. Finally, we used the proposed prescription for finding the propagator concerning Fierz–Pauli higher derivative gravity [7].

This theory is an example of a higher derivative gravity theory which is not gauge-invariant.

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